The Dynamics of Pluralistic Ignorance

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Goal

Model pluralistic ignorance within our Dynamic Social Network Logic framework (DSNL).

- First, model the pluralistic ignorance situation as a specific distribution of (2) properties within a social network.
- Second, represent the change of distribution of these properties within the network given a fixed social influence rule (a model transformation).
- Third, make precise the dynamic properties of PI.

Pluralistic Ignorance state

Informal definition

- All individuals of a group have the same private attitude towards a given proposition p (say a belief in p), but publicly "display" a conflicting attitude towards p (say a belief in $\neg p$).

Two variables

Private vs public sphere

We need to distinguish:

- *Private belief*, “inner belief”
- *Public (or observable) behavior*, “expressed belief”

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- *Private belief*, “inner belief”
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Assumptions

- Agents can only observe the *expressed* beliefs of other agents.
- Default sincerity interpretation: agents interprete the expressed beliefs of others as reflecting their private belief.
- Peer pressure only affects the agents' expressed beliefs

Formally: 2 variables, 3 values each, 6 characteristic propositions

- 2 variables: V_I and V_E.
- Each variable takes one of 3 values: $R_I = R_E = \{Bp, B\neg p, Up\}$

Describing the inner state

- $I_B p := V_I = Bp$: the agent privately believes p
- $I_B \neg p := V_I = B\neg p$: the agent privately believes $\neg p$
- $I_U p := V_I = Up$: the agent is privately undecided about p

Describing the expressed state

- $E_B p := V_E = Bp$: the agent seems to believe that p,
- $E_B \neg p := V_E = B\neg p$: the agent seems to believe that $\neg p$,
- $E_U p := V_E = Up$ the agent seems to be undecided about p.

A 2-property notion of social influence

Influence is represented as a dynamic transformation $\mathcal{I} = (\Phi, \text{post})$.

What influences the behavior of an agent

- His own *inner* state (what he privately believes) AND
- His friends *expressed* states (how his friends behave): the repartition of expressed states among his friends, i.e whether $\langle F \rangle E_B p$, whether $\langle F \rangle E_B \neg p$, and whether $\langle F \rangle E_U p$

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Consequences

- Each agent is in one of 24 possible (relevant) situations. $\mathcal{I}$ assigns a postcondition to each of these 24 situations (dictating what the state of the agent will be next).

Social influence

maximal pressure

- When all of my friends express a belief in p ($\neg p$), I will align and express a belief in p ($\neg p$), whatever my private state is.
- This corresponds to the (1-property) notion of "strong influence" defined by Seligman, Liu and Girard.

weaker pressure

- When not all of my friends express a belief in p ($\neg p$), how I will react depends on what I privately believe about p. But how exactly?
- Several options leading to several different types of agents, or several types of Influence dynamic transformation.

Types of agents

Our type of agents (for now): the most sincere one

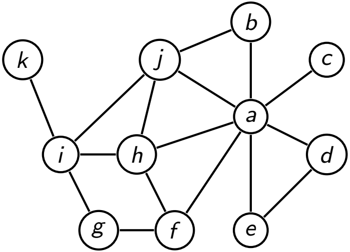
An agent expresses his actual private belief (acts sincerely), iff

- some of his friends expresses the same belief ("support") **OR**
- none of his friends expresses a belief in the negation of what he privately believes ("no conflict")

Pluralistic Ignorance formally

$$PIp := G(I_B p \wedge E_B \neg p)$$

If PIp is true in $\mathcal{M}$ we will say that $\mathcal{M}$ is in a state of pluralistic ignorance with respect to p.



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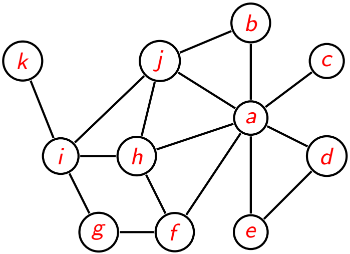

Robustness

Stability of PI

A connected network model in a state of pluralistic ignorance is stable, i.e the following is valid:

$$PIp \to [\mathcal{I}]PIp$$

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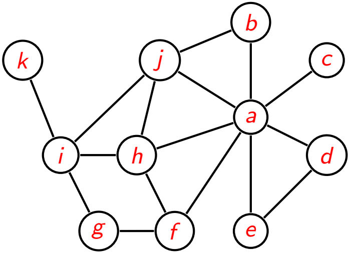


One brave/crazy agent!

Unstable state

Assume one agent i starts being sincere, for some reason, i.e. i expresses his private belief that p:

$$UPIp := @_i(I_B p \wedge E_B p) \wedge G\left(\neg i \rightarrow (I_B p \wedge E_B \neg p)\right).$$

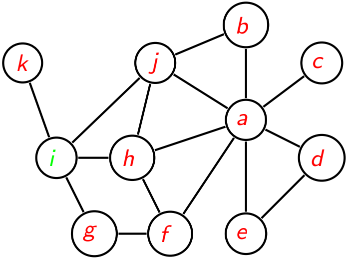
If $UPIp$ is true in $\mathcal{M}$ we will say that $\mathcal{M}$ *is in a state of unstable pluralistic ignorance*.

What happens next? Do all the others start being sincere too?

Breaking PI

- Initially, only i is sincere.

$I_B p \wedge E_B p$ $\quad$ $I_B p \wedge E_B \neg p$



Breaking PI

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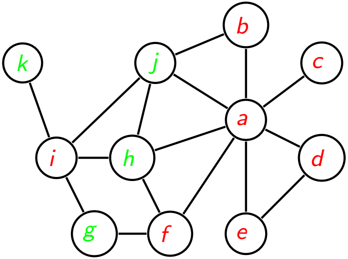

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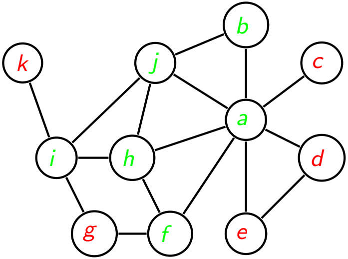

Breaking PI

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- After two steps, all i's friends' friends (including i himself) are sincere.
- After three steps, everybody is sincere.

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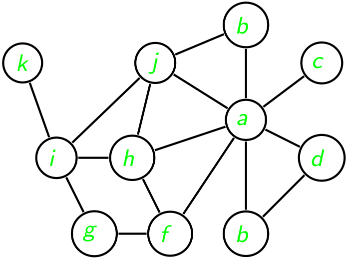

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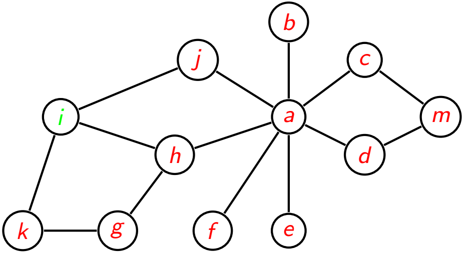

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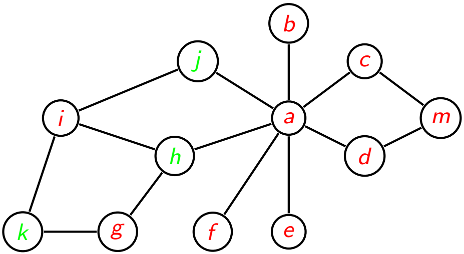

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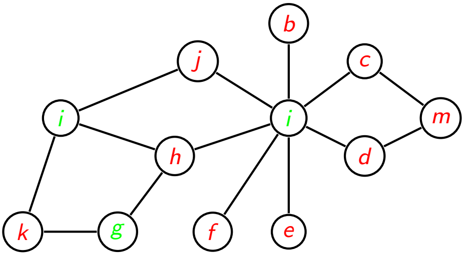

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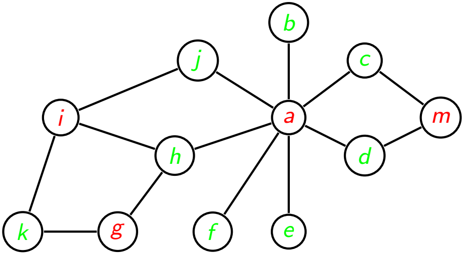

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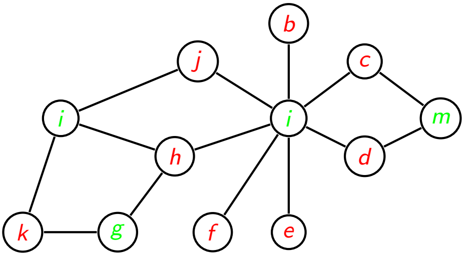

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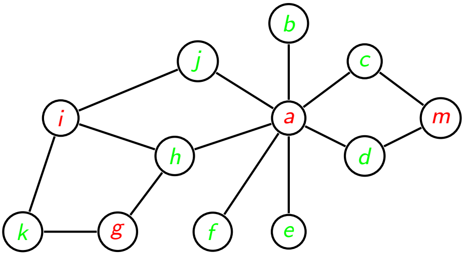

Question 1

When does a model satisfying *UPI* dissolve in a state of global sincerity, i.e such that $G(I_B p \wedge E_B p)$?

Let $\mathcal{M} = (A, \asymp, g, \nu)$ be a finite, connected, symmetric network model such that $\mathcal{M} \models UPI$. Then the following 6 conditions are equivalent:

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When does a model satisfying *UPI* dissolve in a state of global sincerity, i.e such that $G(I_B p \wedge E_B p)$?

Let $\mathcal{M} = (A, \asymp, g, \nu)$ be a finite, connected, symmetric network model such that $\mathcal{M} \models UPI$. Then the following 6 conditions are equivalent:

(i) After a finite number of updates by the influence operator $\mathcal{I}$, $\mathcal{M}$ will end up in a stable state where pluralistic ignorance is dissolved, i.e. there is a $k \in \mathbb{N}$ such that $\mathcal{M}^{\mathcal{I}^k} \models G(I_B p \wedge E_B p)$ and $\mathcal{M}^{\mathcal{I}^k} = \mathcal{M}^{\mathcal{I}^{k+1}}$.

Characterization

iff (ii) There is an agent who expresses her private belief in p for two rounds in a row,
there is an $a \in A$ and a $k \in \mathbb{N}$ such that $\mathcal{M}^{\mathcal{I}^k}, a \models E_B p$ and $\mathcal{M}^{\mathcal{I}^{k+1}}, a \models E_B p$.

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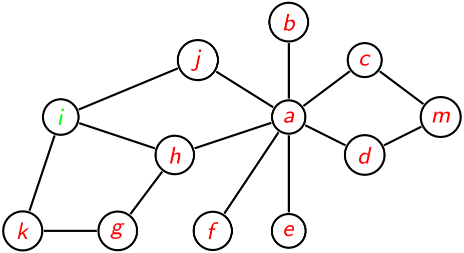
iff (vi) There is a cycle in $\mathcal{M}$ of odd length
there is a $k \in \mathbb{N}$ and $a_1, a_2, \ldots, a_{2k-1} \in A$ such that $a_1 \asymp a_2, a_2 \asymp a_3, \ldots, a_{2k-2} \asymp a_{2k-1}, a_{2k-1} \asymp a_1$.

Example

We have seen an example of network structure which doesn't dissolve from *UPI*. Why? What would need to change to make this possible?



Add a link!

$I_B p \wedge E_B p$ $\quad$ $I_B p \wedge E_B \neg p$

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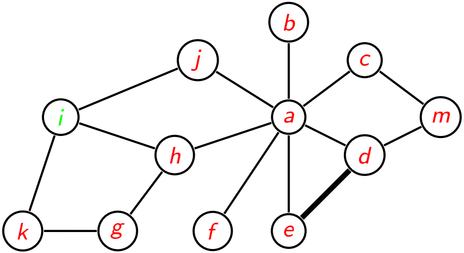
- Create an cycle of odd length from i (v)
- Make two friends have a path of the same length to i (iv)



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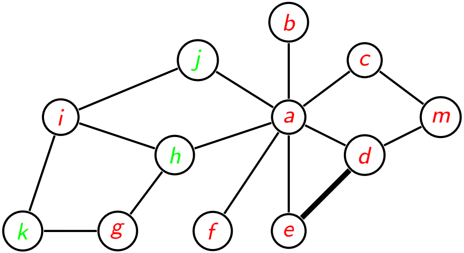

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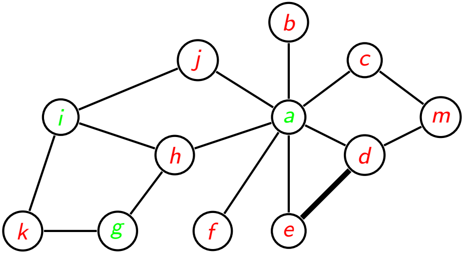

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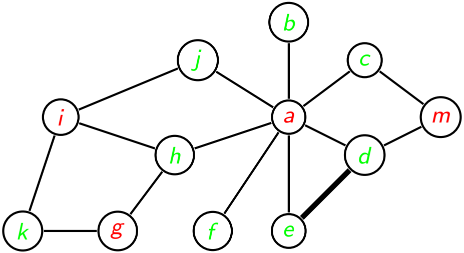
- Make two friends express their private belief at the same round (iii)



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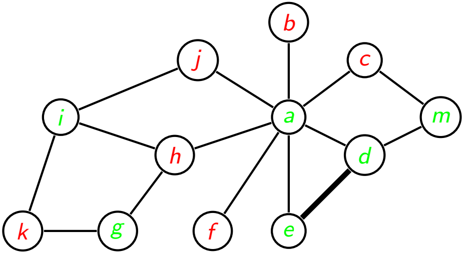
- Make one agent express his private belief for two consecutive rounds (ii)



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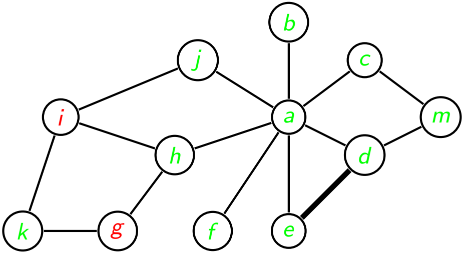

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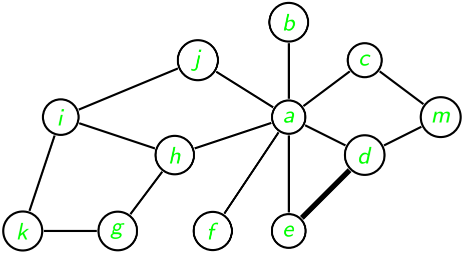
- UPI is dissolved, after 6 applications of $\mathcal{I}$ (i).



What we have done so far

What I have shown you

- Treated an initial example of two-property case dynamics.
- Given a characterization of the class of networks on which a single sincere agent is enough to make sincerity spread to the entire network, in terms of whether the network structure contains an odd cycle.

What I have NOT shown you

- Given an upper bound on how fast a *UPI* model will stabilize.
- Given a characterization of the UPI^n models which stabilize.

Unrealistic?

- The example of looping behavior seems unrealistic.
- If a *UPI* model does NOT stabilize, then no agents (in the entire network) has any friends who are friends with each other.
- That means that the *global clustering coefficient* of such a network is zero.

Further research

- Consider other real life examples (current work with Jens Ulrik Hansen).
- Investigate how modifying the network structure allows/prevents spreading of some combination of properties given different update rules. (current work with Johan van Benthem)
- Use a different framework to model threshold influence. (current work with Alexandru Baltag and Rasmus Rendsvig)
- Investigate the dynamics of different types of agents and mixed networks.

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